1. Linearization & Phase Portraits
: State-Space Representations

System model	state $\underline{x}$, input $\underline{u}$, output $\underline{y}$, time t : $\left.\begin{aligned} \dot{\underline{x}} &= \underline{f}(\underline{x}, \underline{u}, t) \\ \underline{y} &= \underline{h}(\underline{x}, \underline{u}, t) \end{aligned}\right\rbrace \quad (1)$ • Autonomous : (1) doesn't depend on $\underline{u}$. • time-invariant : (1) doesn't depend on t.
→ Control-affine Systems	time-invariant and affine w.r.t $\underline{u}$ $\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u}, t) := \underline{F}(\underline{x}) + G(\underline{x})\underline{u}$

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2: Linearization

Linear. at equil. Point	
• time-invariant dynamics	$\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u})$, $\underline{y} = \underline{h}(\underline{x}, \underline{u})$
• Equilibrium Point [EP]	$\underline{x}^*$, $\underline{u}^*$ with $\underline{f}(\underline{x}^*, \underline{u}^*) = \underline{0} \quad \forall t \geq t_0$
	⇒ Linearization via Taylor series expansion:
	$\dot{\underline{x}}(t) = \dot{\underline{x}}^* + \Delta \underline{x}(t) = \underline{f}(\underline{x}^* + \Delta \underline{x}(t), \underline{u}^* + \Delta \underline{u}(t))$
	$= \underline{f}(\underline{x}^*, \underline{u}^*) + A \Delta \underline{x}(t) + B \Delta \underline{u}(t) + \underline{R}(\Delta \underline{x}^2, \Delta \underline{u}^2)$
Linearized Model ($\underline{R} \approx \underline{0}$)	$\Delta \dot{\underline{x}}(t) = A \Delta \underline{x}(t) + B \Delta \underline{u}(t)$
(small-signal)	$\Delta \underline{y}(t) = C \Delta \underline{x}(t) + D \Delta \underline{u}(t)$
→ Jacobi-Matrix	$\left[\frac{\partial f_i}{\partial x_j}\right] = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}, & \cdots, & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \\ \frac{\partial f_n}{\partial x_1}, & & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$
	$A = \left[\frac{\partial f_i}{\partial x_j}\right](\underline{x}^*, \underline{u}^*)$, $B = \left[\frac{\partial f_i}{\partial u_j}\right](\underline{x}^*, \underline{u}^*)$
	$C = \left[\frac{\partial h_i}{\partial x_j}\right](\underline{x}^*, \underline{u}^*)$, $D = \left[\frac{\partial h_i}{\partial u_j}\right](\underline{x}^*, \underline{u}^*)$

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Linearization about Reference Trajectory	
• time-invariant dynamics	$\dot{\underline{x}} = f(\underline{x}, \underline{u})$, $\underline{y} = h(\underline{x}, \underline{u})$
• Reference trajectory	$\underline{x}^*(t)$, $\underline{u}^*(t)$, $t \geq 0$
	⇒ Linearization via Taylor series expansion: $\dot{\underline{x}}(t) = \dot{\underline{x}}^*(t) + \Delta\dot{\underline{x}}(t) = f(\underline{x}^*(t) + \Delta\underline{x}(t), \underline{u}^*(t) + \Delta\underline{u}(t))$ $= \underline{f}(\underline{x}^*(t), \underline{u}^*(t)) + A(t)\Delta\underline{x}(t) + B(t)\Delta\underline{u}(t) + R(\Delta\underline{x}^2, \Delta\underline{u}^2)$
Linearized model: $\underline{R} \approx \underline{0}$ (small-signal)	$\Delta\dot{\underline{x}}(t) = A(t)\Delta\underline{x}(t) + B(t)\Delta\underline{u}(t)$ $\Delta\underline{y}(t) = C(t)\Delta\underline{x}(t) + D(t)\Delta\underline{u}(t)$
	$A = \left[\frac{\partial f_i}{\partial x_j}\right](\underline{x}^*(t), \underline{u}^*(t))$, $B = \left[\frac{\partial f_i}{\partial u_j}\right](\underline{x}^*(t), \underline{u}^*(t))$ $C = \left[\frac{\partial h_i}{\partial x_j}\right](\underline{x}^*(t), \underline{u}^*(t))$, $D = \left[\frac{\partial h_i}{\partial u_j}\right](\underline{x}^*(t), \underline{u}^*(t))$ time-variant!